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COMPUTATIONAL AND ANALYTICAL INVESTIGATION OF AERODYNAMIC DERIVATIVES OF SIMILITUDE DELTA WING MODEL AT HYPERSONIC SPEEDS

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ABSTRACT

This research paper presents a computational and analytical investigation of aerodynamic derivatives in an oscillating wedge. Unsteady hypersonic similitude has been apprehended for an oscillating wedge with an attached bow shock at a large incidence angle. The problems of instability and shock waves are generally associated with hypersonic flow and, therefore, it is imperative to evaluate aerodynamic models that can solve these problems. Lighthill's piston theory is an unsteady aerodynamic model that is valid for an oscillating wedge with an attached shock wave. The analytical solution verifies that both the stiffness and the damping derivatives attain high values when the semi-vertex angle of the wedge is increased, while both derivatives assume lower values at increasing Mach numbers. Similarly, the pressure distribution over the wedge is evaluated to determine the details of how the developing flow cause the instabilities. Our study presents the contour plots of pressure, temperature, density, and Mach number that unravels the positions of flow separations in an oscillating wedge model.

Keywords: Damping derivatives; Hypersonic flow; Piston theory; Stiffness derivatives; Wedge model

1. INTRODUCTION

There exists a need to research steady, reliable hypersonic technology because of the technology's applications in both commercial and military domains. Some of the pivotal areas of research include space science, space entry vehicles, and advanced long-range weapons. In the domain of space exploration, hypersonic travel is an undeniable necessity; without a good knowledge of hypersonic aerodynamics, future space mission programs will be greatly affected. Recently, there has been a rejuvenation in research into hypersonic flows that is driven by the fascination in developing supersonic aircraft (Huda & Edi 2013; Kuchemann, 2014; Sobieczky, 2014; Morgenstern et al., 2015). One of the key research concerns with high-speed vehicles is the instability phenomenon, which is currently being studied using computational methods (Lamorte & Friedmann, 2014; Lamorte et al., 2014; Mansoorzadeh & Javanmard, 2014; Luo et al., 2015) and by investigating their aerodynamic derivatives (Oppenheimer & Doman, 2006; Corke & Thomas, 2015; Xu & Shi, 2015).

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The augmenting maneuvering capability of modern combat vehicles has emphasized the drawbacks of the conventional stability or aerodynamic derivatives that are based on theoretical models. Important considerations include the stiffness and damping derivatives in pitch for delta wings at high angles of attack during hypersonic flow (Liu et al., 1997; Brandon et al., 2001; Crasta & Khan, 2012). Although these considerations were first recognized in the early 1960s, interest in the determination of dynamic stability derivatives at increasing angles of attack and Mach numbers emerged in the 1970s (Orlik-Rückemann, 1975). Researchers have analyzed the hypersonic flow over wedge cones and flat plates over a range of angles of incidence (Hui, 1969; Pike, 1972; Carrier, 2012; Crasta et al., 2012). The occurrence of steady delta wings at high Mach numbers with attached shock waves has also been studied (Hui, 1969; Pike, 1972). Exact solutions have been obtained for 2-Dimensional flows for an oscillating wedge and flat plate, which have proved valid for all high Mach numbers and wedge angles of attack with attached shock.

The piston theory was originally developed for oscillating airfoils at supersonic and hypersonic speeds (Lighthill, 2012). Ghosh (1984) obtained a similitude and two analogous characteristics for an edge shock attached to oscillating delta wings at high incidence. Ghosh's similitude to supersonic/hypersonic flows past a planar wedge was extended in a study by Crasta and Khan (2013) and used to obtain stability derivatives in the pitch and roll of a delta wing (Crasta et al., 2012). This investigation is the extension of work by Crasta and Khan (2014) who were concerned with the pressure and shock distributions of the hypersonic flow around an oscillating wedge at different angles of attack.

Our concern is with the aerodynamic derivatives of hypersonic flow around a wedge of a small angle and the pressure distribution around the similitude delta wing. We will concentrate on the analytical solution derived by Crasta and Khan (2014), and we will also present a computational analysis. In the related calculation, Ghosh's similitude model at hypersonic speeds is considered. Hence, the aim of this study is to examine the flow effects using computational analysis for the viscous wedge model.

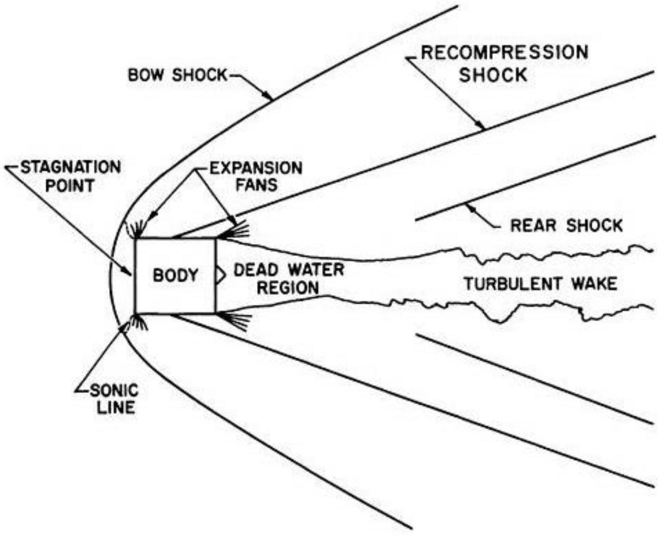

Figure 1 Sketch of hypersonic flow field on a flat surface at M = 5

2. GEOMETRY MODEL AND OPTIMIZATION PROCESS

Two-dimensional wedge geometry is used as a model for this study. The angle of incidence was constant at 25° and the chord length of the wedge was fixed at 1 m. Figure 2 shows the geometry of the wedge, and Table 1 summarizes the dimensions of the model.

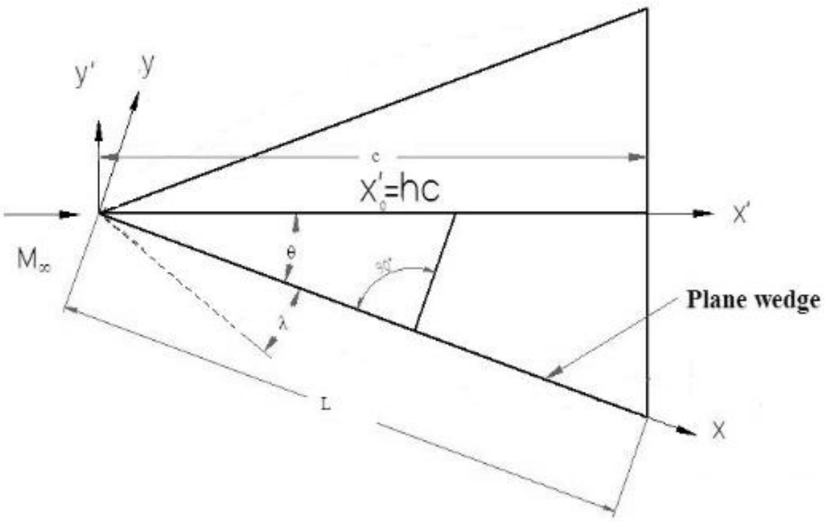

Figure 2 2-D geometry of the wedge

From Figure 2, we obtain:

$$c = L\cos\theta$$
$$x_o = x_o'\cos\theta$$
$$Define\ h = {}^{x}/_{c}$$
$$\therefore\ x_o' = h_o c = h_o L\cos\theta$$
$$\therefore\ x_o = h_o L\,(\cos\theta)^2$$

With the changes in the equations of the analytical solution model, the correct equations for the stiffness and damping derivatives of a wedge can be written as:

$$-c_{m_\alpha} = (\gamma+1)\sin\theta\cos\theta\left[2 + \sqrt{\left(\frac{4}{\gamma+1}\right)^2 + M_1^2(\sin\theta)^2}\Big/ M_1\sin\theta + M_1\sin\theta\Big/\sqrt{\left(\frac{4}{\gamma+1}\right)^2 + M_1^2(\sin\theta)^2}\right]\times\frac{1}{c^2}\left[\frac{L^2}{2} - h_o L^2(\cos\theta)^2\right] \tag{1}$$

$$-C_{m_q} = (\gamma+1)\tan\theta/(\cos\theta)^2 \left[2 + \sqrt{\left(\frac{4}{\gamma+1}\right)^2 + M_1^2(\sin\theta)^2} \Big/ M_1\sin\theta + M_1\sin\theta \Big/ \sqrt{\left(\frac{4}{\gamma+1}\right)^2 + M_1^2(\sin\theta)^2}\right] \times \left(\frac{1}{3} - h_o(\cos\theta)^2 + h_o^2(\cos\theta)^4\right) \tag{2}$$

Table 1 Dimensions of the model

Parameter	Value
Angle of incidence (θ)	25°
h = *x*/*c*	1 m
Height	1.5 m

For the general solution setup, a density-based solver has been chosen since the flow is considered to be compressible and the energy equation is involved in the solver.

For the viscous model of the solver, the k-epsilon model has been chosen instead of the laminar model since this simulation is for hypersonic flow. The k-epsilon model is one of the most common turbulence models. It is a two-equation model, which means that it includes two extra transport equations to represent the turbulent properties of the flow. A two-equation model accounts for reported effects such as the convection and diffusion of turbulent energy.

3. RESULTS AND DISCUSSION

We investigated the variation of the stability derivatives with a pivot position for various Mach numbers and angles of incidence. Both the computational and analytical studies have been considered, and the stiffness and damping derivative have been compared with these earlier results. We will use contour plots to determine the pressure and temperature distributions at various sections of the wedge model. In the analytical solution, the stiffness derivative shows good agreement with the results presented by earlier theories such as Lighthill's theory. The results are discussed in the following sections.

3.1. ANSYS CFD Simulation over the Wedge for Mach number 7

In this work, an ANSYS Computational Fluid Dynamics (CFD) simulation was performed for Mach number 7. The results of the simulation are shown in Figures 3, 4, and 5. Figure 3 shows the pressure distribution along the surface of the wedge. Since we are using a planar wedge at a 25° angle of incidence, the pressure distribution trend for all cases viz. different Mach numbers are almost identical. The pressure seems to be increasing toward the aft position of the wedge, and the shockwave angle is observed. The sudden variations in the pressure distribution at high Mach numbers results in the formation of this shock angle.

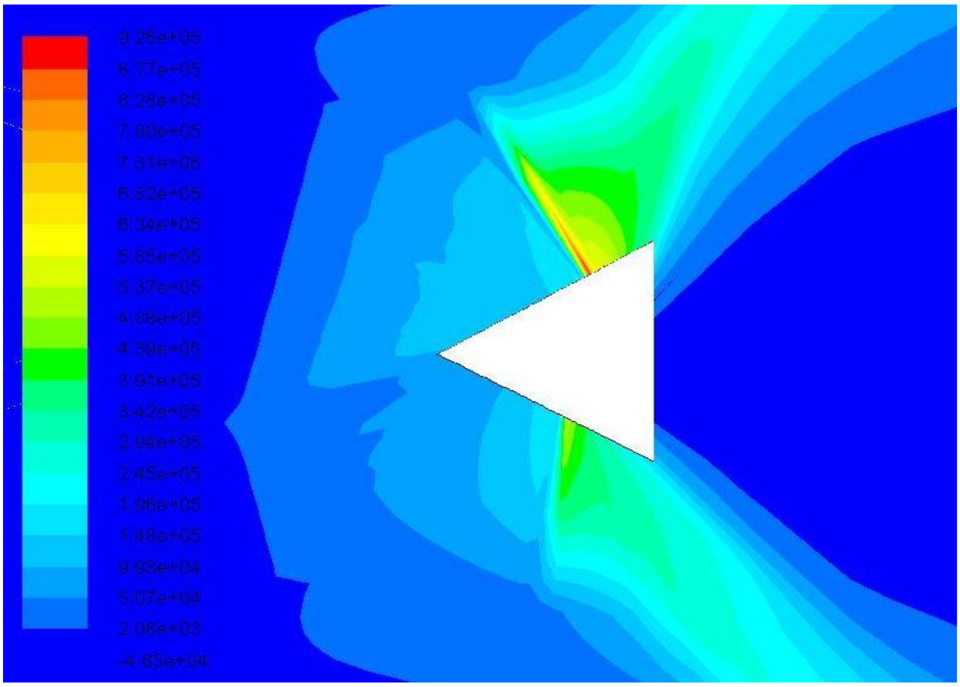

Figure 3 Pressure contour over the wedge at M = 7

Figure 4 depicts the velocity contour of the simulation model; the flow at the surface of the body has a lower velocity, while flow separation occurs at the aft position of the wedge model. Figure 5 depicts the temperature contour of the model; the temperature is higher near the body surface and at the rear surface of the body. This is attributed to the formation of high-speed vortices at the rear of the body, and the heat generated due to skin friction drag with the body surface.

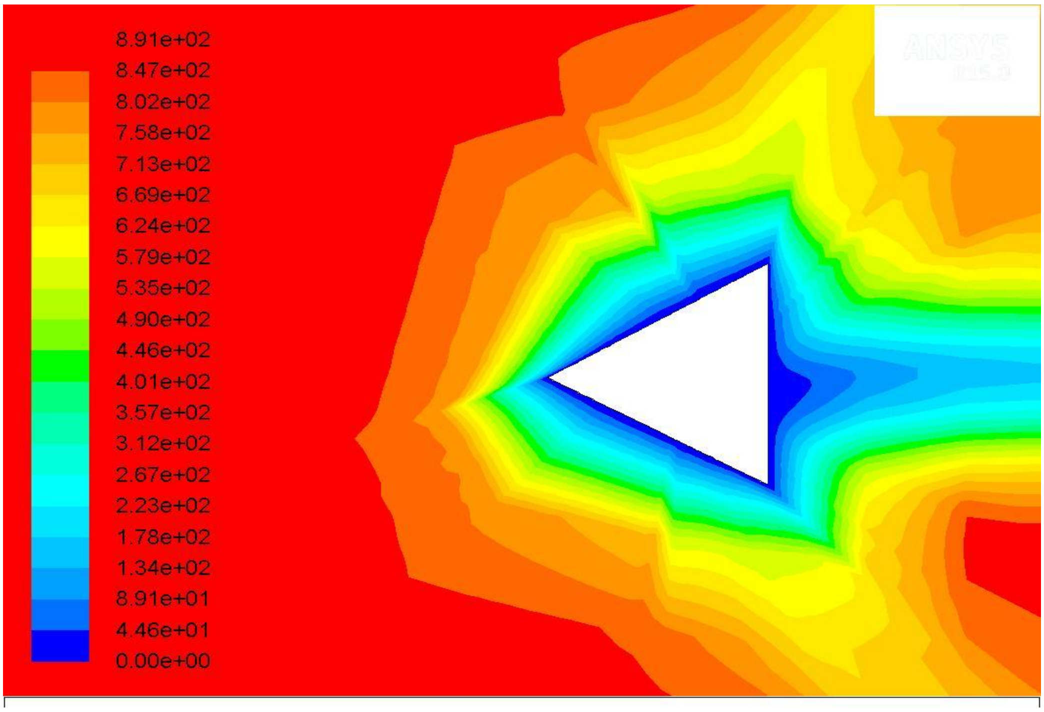

Figure 4 Velocity contour over the wedge at M = 7

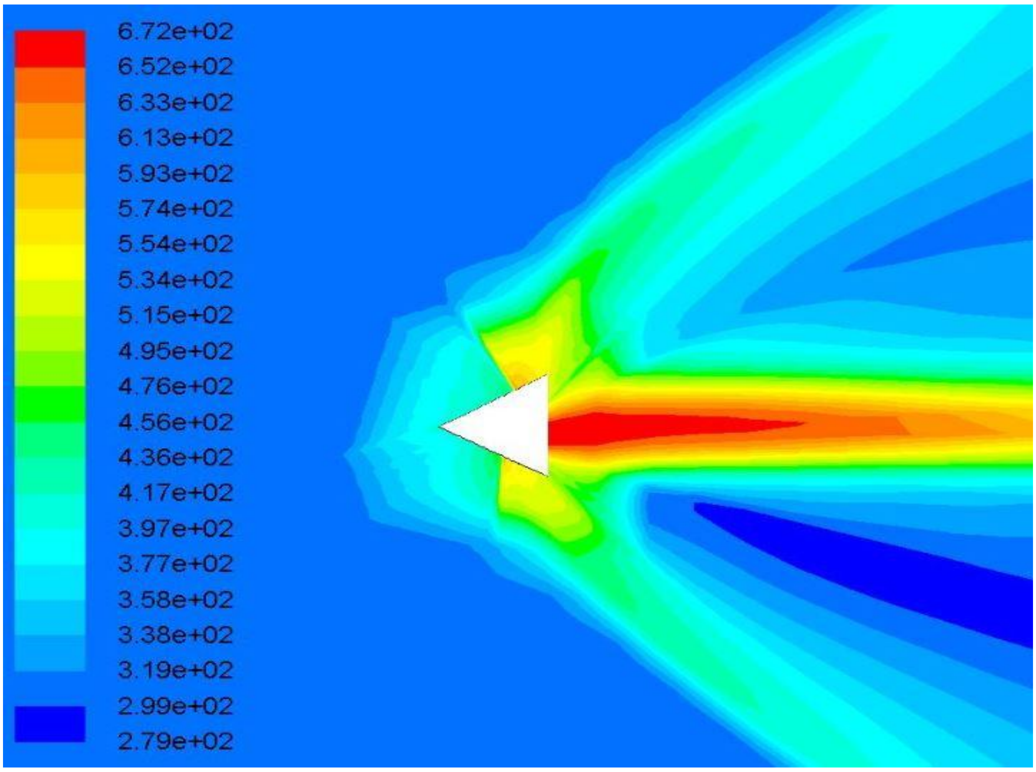

Figure 5 Temperature contour over the wedge at M = 7

3.2. Comparison of Lighthill's Theory with the Present Theory

The three theories for obtaining the pressure ratio are shown in Figures 6 and 7 for Mach numbers 12 and 17, respectively. It is clearly shown that the pressure ratio is valid up to a 15° angle of incidence. The figures show that Lighthill's curve rapidly moves away from the oblique shock relation, while the present theory shows a good accuracy when compared to oblique shock relation. The accuracy of Lighthill's piston theory will gradually decrease with the increase of the flow Mach number because of the shrinkage of Lighthill's region.

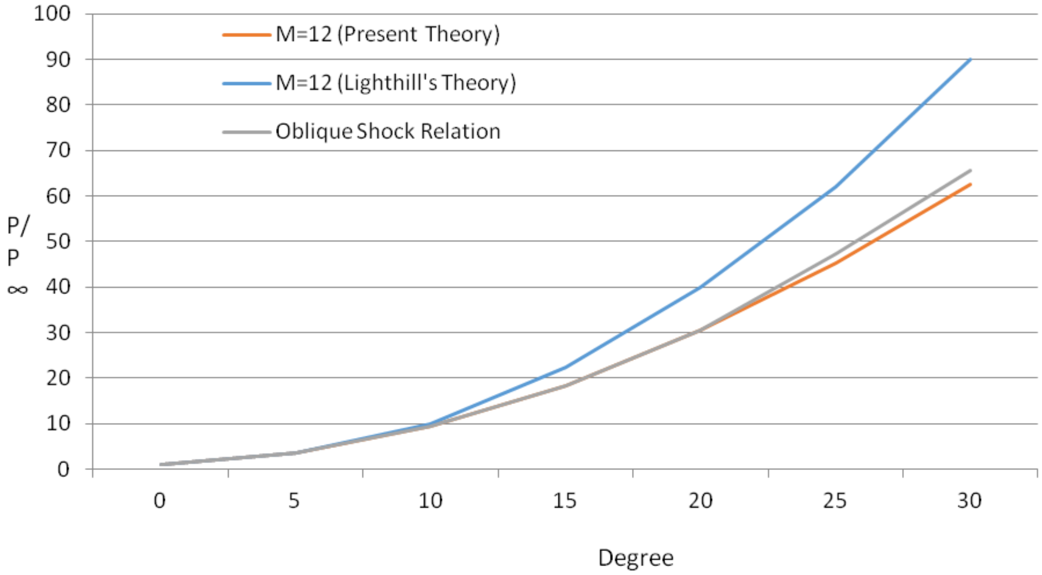

Figure 6 Comparison graph of pressure ratio vs. angle of incidence for M = 12

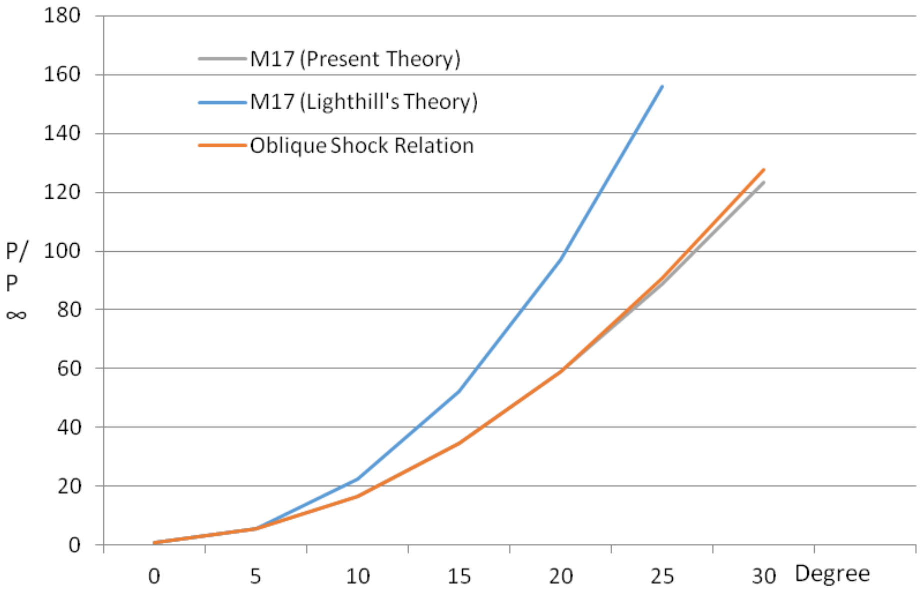

Figure 7 Comparison graph of pressure ratio vs. angle of incidence for M = 17

The stiffness derivatives were also compared with the results of the oblique shock theory as shown in Figure 8. A good agreement is obtained for the stiffness derivative for a lower semi-vertex angle θ, as well as for Mach number 17; note that only the theoretical and experimental values were available for this Mach number. The error does not seem to exceed 10%. However, for the damping derivative, the difference is noticeable, particularly for a very high flow deflection angle as shown in Figure 9.

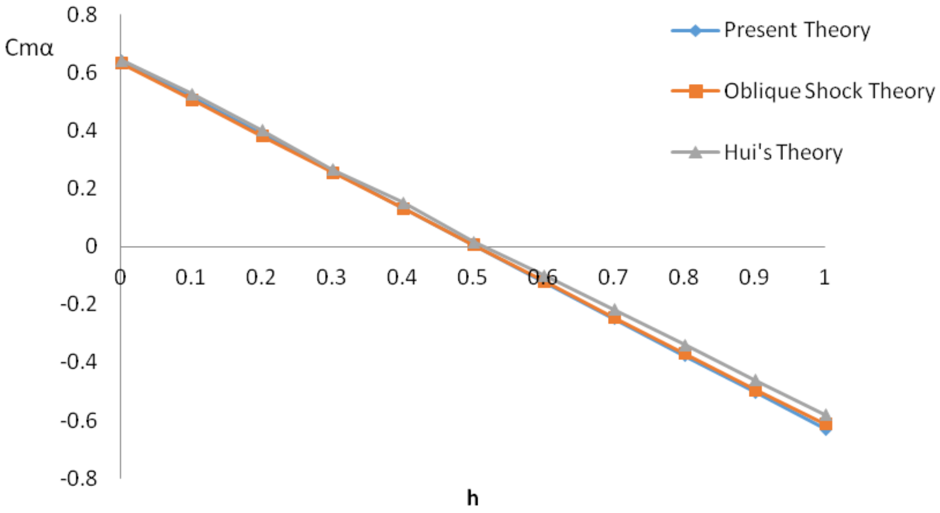

Figure 8 Comparison graph of stiffness derivative vs. pivot position for M = 17

3.3. Analytical Method for Stiffness and Damping Derivatives at Different Mach Numbers

The stiffness and damping derivatives with respect to the Mach number for a different angle of incidence at a pivot position are shown in Figures 10 and 11, respectively. The trends are identical. The Mach number independent principle is clearly demonstrated here. As we know, when the Mach number increases, the shock wave angle decreases. However, after a certain Mach number, there is no decrease in the shock wave angle, which implies that an initial increase in Mach number causes the shock to come very close to the body, although it will never attach to the body.

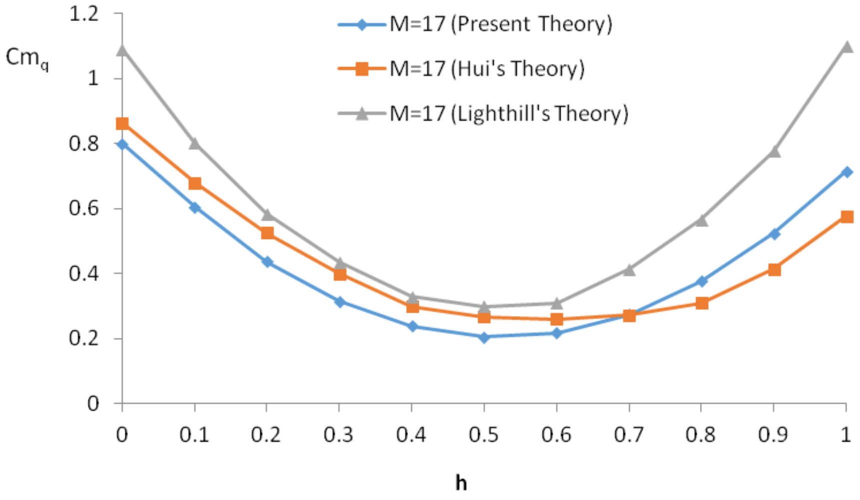

Figure 9 Comparison graph of damping derivative vs. pivot position for M =17

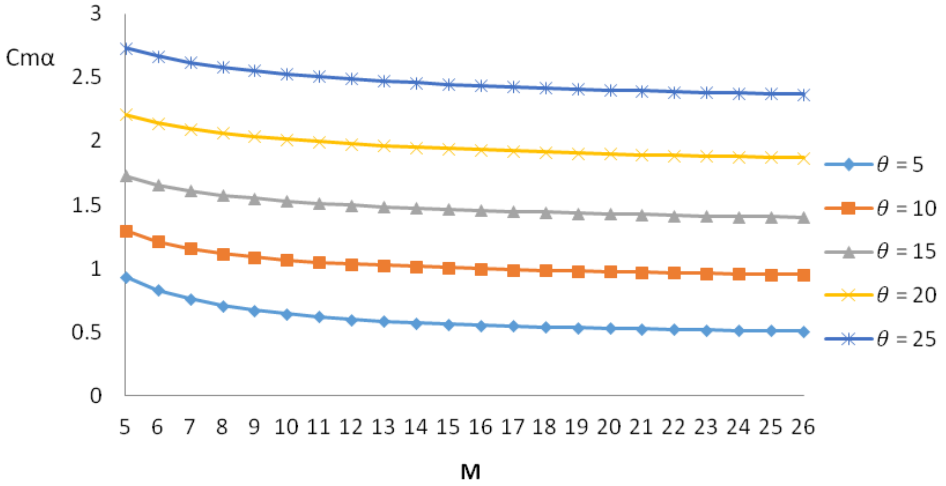

Figure 10 Graph of stiffness derivative vs. angle of incidence at various Mach numbers

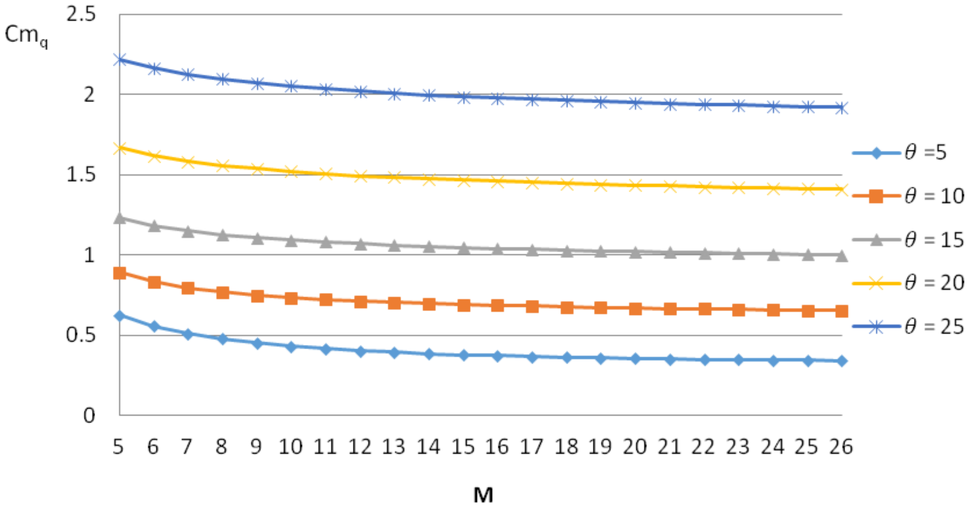

Figure 11 Graph of damping derivative vs. angle of incidence at various Mach numbers

4. CONCLUSION

Based on the analytical method, it is evident that as the angle of attack increases at h = 0.0, there is a continuous increase in the values of the damping derivatives. Our results also showed that the stiffness derivative decreases linearly with the pivot position. In addition, the stiffness derivatives linearly increase with an increase in angle of incidence; this should occur theoretically, and the trend was expected. Furthermore, from the graphical analysis, we can observe that an increase in the angle of incidence causes a continuous shift of the center of pressure toward the aft position of the wedge. Due to this phenomenon, the method in which the angle of incidence is continuously increased could be used to stabilize aerodynamic vehicles. For some cases, this would mean that the requirement of a large stabilizing surface could be avoided. This theory is valid only when the shockwave is attached to the nose of the wedge.

For the simulation method, we observed that the pressure distribution is significantly large at the back of the wedge and it increases abruptly as the Mach number increases up to 20. This is due to the high drag induced by the rear of the wedge. Thus, at this stage, aerodynamic heating is the main concern since extreme temperatures could lead to crucial aerodynamic fatality.

To compute the stability derivatives from the CFD data with more accuracy, a detailed 3-Dimensional geometric model need to be investigated, and calculation made in the future. To achieve a good comparison, a study using different software should also be done. Finally, the full cooperation between the supervisor and project partner is the key element for the project to succeed.

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